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The paper addresses a new observational method that allows one to compute the astronomic zenith distance of the astrolabe instrument as a function of the time when the stars cross the almucantar of the astrolabe and their right ascension and declination, therefore independent of the astronomic station coordinates.

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Astrogeodetic Deflections of the Vertical From Stars Observations with the Danjon Astrolabe, or Similar Instruments

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BIOGRAPHICAL SKETCH

Dr. Angel A. Baldini joined ETL's predecessor organization in 1960. From 1957-1960 he was associated with the Georgetown University Observatory, Washington, D.C. Prior to 1957, he was Professor and Head, Department of Geodesy, La Plata University, Argentina. At ETL, Dr. Baldini works primarily in the field of astro-geodesy. Since 1974 he has been senior scientist in the Center for Geodesy, Research Institute, ETL. He authored 45 reports and papers since 1963 and presented the results at 25 Army, national, and international meetings. He received an Army Research Development achievement award in 1969. He has been a member of the American Geophysical Union since 1960.

ABSTRACT

This report is a sequel to the report "Basic Formulae for Determining the Computation of the Deflection of the Vertical from Astronomic Star Observations," presented at the 1985 ASP-ACSM Convention, Washington, DC. (AD-A154 112), 34

The paper addresses a new observational method that allows one to compute the astronomic zenith distance of the astrolabe instrument as a function of the time when the stars cross the almucantar of the astrolabe and their right ascension and declination, therefore independent of the astronomic station coordinates. The ξ and η deflection of the vertical components are obtained from an equation that relates the astronomic and geodetic zenith distance. The last is derived for each star as a function of the U.T.1 time and geodetic latitude and longitude.

INTRODUCTION

The first part of this paper deals with the definition of a point on a sphere as a common vertex of isosceles triangles, when the position of great circle arcs among the points on the sphere are known. One application of this concept is in astronomy of position. Herein it is applied to astrolabe observations to obtain the instrument constant zenith distance. Many isosceles triangles, depending on the number of stars used, can be considered. A constraint is that their common vertex is the zenith. In this particular case the almucantar zenith distance is obtained. For the second part of this report, vertical deflection components are obtained from astronomic, and the geodetic zenith distance of each observed star with respect to UT1 Time. Astronomic latitude and longitude may be obtained thereafter. A computation example is treated in some detail, employing data generated by a Danjon Astrolabe at the US Naval Observatory, Washington, D.C.

FUNDAMENTAL CONCEPTS

Consider a sphere of unit radius, with center at C, and three points 1, 2, 3 on it. Let these points be joined with great circles of arc each of which pass through two points, as shown in Figure 1, and let a portion of the great circles of arc be:

$$a = 1.2 \qquad b = 2.3 \qquad c = 1.3 \tag{1}$$

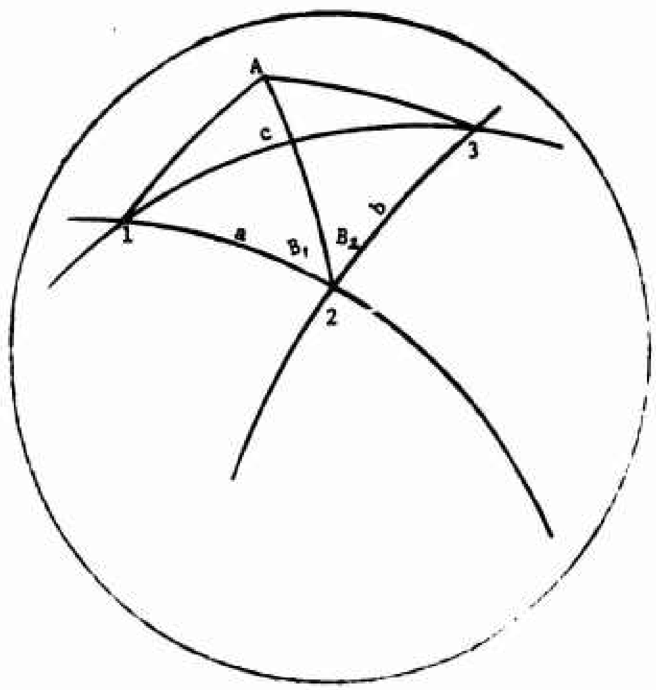

Figure 1.

On this sphere let point A be chosen in order to form two isosceles triangles. The two triangles are: 1-2-A and 2-3-A.

Let the angles at point 2, be:

$$B_1 = 1\text{-}2\text{-}A \qquad B_2 = 3\text{-}2\text{-}A \tag{2}$$

If the arcs a, b, and c, are known, the angles B_1 and B_2 can be computed as a function of them, hence the point A can be defined. Consequently the portion of the great arcs between A and the points 1, 2, and 3, must be equal. The sum of (B_1 + B2) can be known through the equation

$$\cos(B_1 + B_2) = \frac{\cos c - \cos a \cos b}{\sin a \sin b} \tag{3}$$

We have found that theses angles can be obtained from the equations

$$\tan B_1 = \frac{\tan \frac{1}{2}b - \tan \frac{1}{2}a \cos(B_1+B_2)}{\tan \frac{1}{2}a \sin(B_1+B_2)} \tag{4}$$

$$\tan B_2 = \frac{\tan \frac{1}{2}a - \tan \frac{1}{2}b \sin(B_1+B_2)}{\tan \frac{1}{2}b \sin(B_1+B_2)} \tag{5}$$

To obtain the arc distance from A to the points 1, 2, and 3 we have:

$$y = A\cdot 1 = A\cdot 2 = A\cdot 3$$

then y can be obtained from the following equations:

$$\tan y = \frac{\tan \frac{1}{2}a}{\cos B_1} = \frac{\tan \frac{1}{2}b}{\cos B_2} \tag{6}$$

Let us consider an application of this fundamental concept to astrolabe observations.

APPLICATION OF THE FUNDAMENTAL CONCEPT TO ASTROLABE OBSERVATIONS

The main application of the preceeding concept is for observations of stars at an equal altitude, because many isosceles triangles can be considered. This is the case when the Danj n Astrolabe or similar instrument is employed. A theodolite may be used in the same way as an astrolabe, provided that during observations no movement should change the angle between the telescope and the bubble. To this case point 1, 2, 3, are replaced by images of the stars and the point A will be the zenith, the pole of the horizon. The arcs a, b, c, are related to the meridian of the stars and the astronomical meridian. The arc y then will be the almucantar zenith distance. If we observe more than three stars at the same altitude, we have more than sufficient data for the determination of the almucantêr zenith.

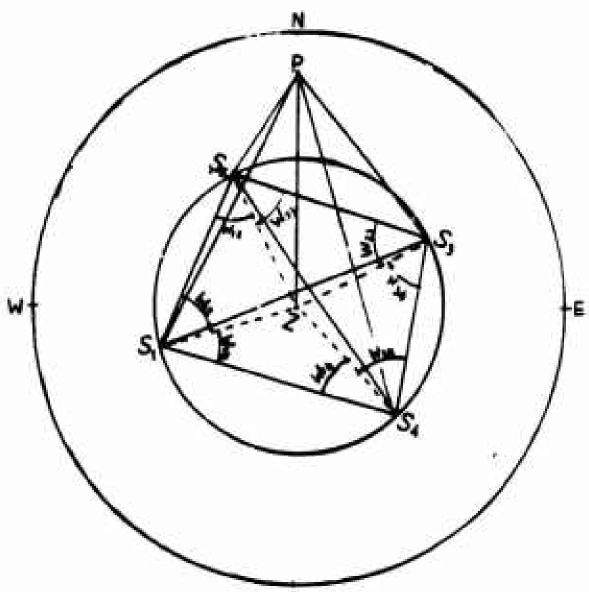

Figure 2. Unit Sphere Projected from Above.

$$\sigma_{12} = (T_2 - T_1)(1+c) - (\alpha_2 - \alpha_1) = S_1 P S_2$$
$$\sigma_{13} = (T_3 - T_1)(1+c) - (\alpha_3 - \alpha_1) = S_1 P S_3$$
$$\text{- - - - - - - - - - - - - -}$$
$$\sigma_{34} = (T_4 - T_3)(1+c) - (\alpha_4 - \alpha_3) = S_3 P S_4 \qquad (7)$$
$$c = 0 \text{ if the clock is regulated to sidereal time}$$
$$c = 0.002737909 \text{ for mean time}$$

The arc of the great circle that passes between two stars is computed from the general equation:

$$\cos S_{ij} = \sin \delta_i \sin \delta_j + \cos \delta_i \cos \delta_j \cos \sigma_{ij} \qquad (8)$$

Let us designate by the angle of the isosceles triangles as seen in figure 2. The zenith distance then can be computed according to equation (7).

Let us now find out the relationship of a star position to the astronomical and geodetic points, to obtain the deflections of the vertical as functions of the astronomical and geodetic star zenith distance.

DEFLECTIONS OF THE VERTICAL AS FUNCTION OF THE ASTRONOMIC AND GEODETIC ZENITH DISTANCES OF A STAR.

For this purpose we use a general formula that links a star position to its zenith distance and hour angle, setting it as follows in the astronomical system:

$$2 \frac{\cos Z}{\cos \delta} = 2 \tan \delta \sin \phi + \cos(\phi + t) + \cos(\phi - t) \qquad (9)$$

A similiar equation can be set for the station in the local geodetic system as follows:

$$2 \frac{\cos \zeta}{\cos \delta} = 2 \tan \delta \sin B + \cos(B + \tau) + \cos(B - \tau) \qquad (10)$$

Z, ζ = astronomic and geodetic zenith distance
ϕ, B = astronomic and geodetic latitude
t, τ = astronomic and geodetic hour angle
δ = star's declination

We have to remember that ϕ and t are unknowns, while B and L are knowns, but they differ, with respect to ϕ and λ , by small amount, ξ and $\eta \sec \phi$, the deflections of the vertical. From equations (9) and (10) we shall derive an equation for obtaining ξ and η as a function of the astronomic and geodetic zenith distance. Subtracting equation (10) from (9) we have

$$\frac{-4 \sin \frac{1}{2}(Z - \zeta) \sin \frac{1}{2}(Z + \zeta)}{\cos \delta} = \begin{array}{l} 4 \tan \delta \sin \frac{1}{2}(\phi - B) \cos \frac{1}{2}(\phi + B) \\ + \cos(\phi + t) + \cos(\phi - t) - \cos(B + \tau) - \cos(B - \tau) \end{array} \qquad (11)$$

Now let

$$X_1 = \frac{1}{2}(\phi + t)$$
$$X_2 = \frac{1}{2}(B - \tau)$$

so it can be set

$$\frac{1}{2}(X_1 + X_2) = \frac{1}{2}(\phi + B + \frac{1}{2}(t - \tau)$$
$$\frac{1}{2}(X_1 - X_2) = \frac{1}{2}(\phi - B) + \frac{1}{2}(t + \tau)$$

then it follows at once that the corresponding terms on the right hand side of equation (11) can be expressed as follows:

$$\cos\chi_1 - \cos\chi_2 = -2\left[\left(\sin\left(\frac{\phi+B}{2}\right)\cos\left(\frac{t-\tau}{2}\right) + \cos\left(\frac{\phi+B}{2}\right)\sin\left(\frac{t-\tau}{2}\right)\right)\left(\sin\left(\frac{\phi-B}{2}\right)\cos\left(\frac{t+\tau}{2}\right) + \cos\left(\frac{\phi-B}{2}\right)\sin\left(\frac{t+\tau}{2}\right)\right)\right] \quad (12)$$

Due to the small values of $\frac{\phi-B}{2}$ and $\frac{t-\tau}{2}$, we set for practical purposes:

$$\cos\frac{\phi-B}{2} = \cos\frac{t-\tau}{2} = 1$$

hence equation (12) becomes:

$$\cos\chi_1 - \cos\chi_2 = -(\phi-B)\sin B\cos\tau - 2\sin\left(\frac{\phi+B}{2}\right)\sin\left(\frac{t+\tau}{2}\right) - (t-\tau)\cos B\sin\tau \quad (13)$$

In a similiar way, let

$$m = \phi - t \qquad \frac{m+n}{2} = \frac{\phi+B}{2} - \frac{t-\tau}{2}$$

$$n = B+\tau \qquad \frac{m-n}{2} = \frac{\phi-B}{2} - \frac{t+\tau}{2}$$

from which it is obtained for the third and fourth term on the right side of equation (11)

$$\cos m - \cos n = -(\phi-B)\sin B\cos\tau + 2\sin\frac{\phi+B}{2}\sin\frac{t+\tau}{2} - (t-\tau)\cos B\sin\tau \quad (14)$$

adding to equation (11), equations (13) and (14), and using the constraints

$$\begin{aligned} \phi - B &= \xi \\ t - \tau &= L - \lambda = \eta\sec B \end{aligned} \quad (15)$$

as well as the fact that $(Z-\zeta)$ is small, we obtain the final expression for computing the ξ and η deflections of the vertical components:

$$(Z-\zeta)\frac{\sin Z}{\cos\delta} = \xi(\tan\delta\cos B - \sin B\cos\tau) + \eta\sin\tau \quad (16)$$

where

$$\tau = UT1(1+c) + \theta_0 - (\alpha - L) \quad (17)$$

and

UT1 = universal time
θ_0 = apparent sidereal time at 0^h U.T.
L = geodetic longitude
B = geodetic latitude
ζ = geodetic zenith distance
Z = astronomic zenith distance

The geodetic zenith distance is known by computation from

$$\cos\zeta = \sin B\sin\delta + \cos B\cos\delta\cos\tau \quad (18)$$

Each star provides an observation equation of the form:

$$a.\xi + b.\eta = (\zeta - Z)\sin Z\sec\delta$$

hence the four stars will give four equations for and .
Choosing star pair clockwise, four values of ξ and η can be obtained.

As an example the following data taken with a Danjon Astrolabe at the U.S. Naval Observatory, Washington, D.C., on August 17, 1984, was used to compute the deflections of the vertical.

STAR	NUMBEIUT FK4	TIME OF ALMO OBSERVED			RT. ASC.			CFCL.		
		H	M	S	H	M	S	0	/	//
1	774	2	39	49.4225	20	38	53.001	15	51	15.267
2XX	3388	2	43	20.3393	17	26	6.534	20	5	50.537
3	1434	2	46	20.7368	16	38	18.195	48	57	55.052
4	778	2	48	42.1587	20	42	42.012	15	0	56.687
5	3383	2	51	44.8365	17	23	25.311	22	58	41.321
6	1565	2	55	58.9911	21	29	12.801	23	33	59.647
7	1469	3	0	35.0479	17	59	19.591	16	45	12.939
8	804	3	2	47.8230	21	21	20.139	19	44	2.667
9	1465	3	6	51.2833	17	47	42.502	20	34	23.096
10	3669	3	10	0.1091	20	54	50.470	13	39	32.150
11	844	3	14	35.1432	22	22	55.954	52	8	42.027
12	848	3	20	24.2235	22	30	37.854	50	11	46.688
13	3381	3	24	58.3336	17	21	24.605	53	26	25.224
14	831	3	27	26.1903	22	6	15.300	25	15	51.258
15	685	3	30	33.8042	13	13	49.930	64	23	43.817
16	650	3	34	29.4979	17	26	18.567	48	16	40.425
17	725	3	40	57.7886	19	17	3.024	11	33	59.565
18	701	RVED			18	36	12.334	65	28	38.433
19	3833	3	45	10.6755		56	21.704	48	35	30.229
20	663	3	47	25.0697	17	39	0.539	46	1	7.768
21	1503	3	51	26.9137	19	24	11.525	11,54		35.615
22XX	1604	3	56	5.0920	23	7	1.315	49	12	14.684
23	3839	3	59	58.6334	22	59	24.577	56	51	16.941
24	703	4	3	46.8881	18	44	57.579	20	31	55.612
25	3514	RVED			18	56	23.592	65	14	20.212
26	853	4	10	33.3777	22	38	5.861	63	29	47.948
27	875	4	13	38.4420	23	12	30.443	57	4	30.110
28	826	4	19	58.0237	22	0	17.769	13	2	27.240

The set of four stars used in this example, according to its azimuth is: 5-16-12-4, which we reorder with numbers: 1-2-3-4. Geodetic coordinates, arbitrarily chosen, are

$$B = 39^{o}\ 54'\ 00$$
$$L = -5^{h}\ 08^{m}\ 13^{s}.576$$

Table 1, shows the evaluation of zenith distance of the Danjon Astrolabe. The first column indicates the two angles oppossed to the zenith, as shown in Figure 2, column 2, the corresponding values of these angles are computed according to equation (3). Column 3, 4, 5, and (6), show the values of the angles W_{ij} computed according to equation (4).

Table 2, shows the values of the geodetic zenith distances computed by using equation (16). The geodetic hour angle τ , is obtained from (17), the values of which are shown in column 2. In column 3, are the values of Z_i and in the fourth column the difference between geodetic and astronomic zenith distance.

Table 3, shows the values of the coefficients a, b, c, computed from:

$$
\begin{aligned}
a &= \tan\delta\cos\beta - \sin\beta\cos\tau \\
b &= \sin\tau \\
c &= \sin z/\cos\delta
\end{aligned}
$$

In the last column are the values of the left member of equation (16).

Table 4, shows the evaluation of the deflection components according to equation (16). The evaluation of ξ and η are computed from the equation:

$$a \cdot \xi + b \cdot \eta = c\,(\zeta - Z)$$

taken clockwise we use two equations, in the following order: 1-2; 2-3; 3-4; 4-1.

Table 1. Evaluation of Astronomic Zenith Distance

Reference Angle	Evaluation Angle	W_{12}	W_{14}	W_{23}	W_{34}
$W_{12} + W_{23} =$	103° 03' 31.88	65° 55' 10"75		37° 08' 21"13	
$W_{12} + W_{14} =$	105 03 31.64	65 55 11.10	39 08 20.54		
$W_{14} + W_{34} =$	92 02 12.92		39 08 19.71		52' 53' 53.21
$W_{23} + W_{34} =$	90 02 15.52			37 08 21.95	52 53 53.57
Mean		65° 55' 10.93	39° 08 20"13	37° 08 21"54	52° 53' 53.39
S		23 31 07.59	48 15 33.37	49 26 29.94	38 24 54.
Z		30° 00' 27"76	30° 00' 27".79	30° 00' 27".72	30° 00" 27".75
Mean			Z = 30° 00" 27".75		

Table 2. Evaluation of Geodetic Zenith Distance and

STA	τ	ζ	$\zeta - Z$
1	29° 56" 56".09	30° 00' 25".28	-2"47
2	39 56 32.50	30 01 34.59	66.84
3	-39 40 10.64	30 00 47.21	19.75
4	-20 38 0210	29 59 10.04	-71.75

Table 3, Evaluation of Coefficients a, b, c

a	b	c	$c(\zeta - Z)$
-0.2121	0.4992	0.5432	-1"34
0.3913	0.6428	0.7515	50.23
0.4506	-06384	0.7812	15.43
-0.3709	-03524	0.5178	-40.24

Table 4. Evaluation of Deflection of the Vertical

Station Pair Clockwise	ξ	η
1-2	77"75	30.82
2-3	77.83	30.76
3-4	77.68	30.66
4-1	77.60	30.75

The ξ and η values are large because an arbituary geodetic point was used.

CONCLUSIONS

The classical method of determining deflections of the vertical compares astronomic and geodetic latitudes and longitudes from the relations.

$$\xi = \varphi - B$$

$$\eta = (\lambda - L) \cos B$$

By astronomic observations, latitude φ and longitude λ, are determined. The corresponding geodetic values B and L, are known from geodetic measurements.

The observational method developed in this paper with a solution for vertical deflections from astronomic and geodetic zenith distance can be employed advantageously By contrast with the classical method, this method does not require the knowledge of the astronomic coordinates, hence reducing the time and cost of field work.

Results from several independent sets of four stars have shown that the determination of the almucantar zenith distance is less prone to errors than for the determination of latitude and longitude. If astronomic coordinates are needed, then the geodetic values B and L can be used as an approximate value of latitude φ and longitude λ. The and values must be replaced by

$$\xi = d\varphi$$
$$\eta = d\lambda \cos B$$

hence, the astronomic station coordinates shall be

$$\varphi = B + \xi$$
$$\lambda = L + \eta \sec B$$

The ξ and η values must be corrected for the effect of polar migration, as follows

$$d\xi = -x \cos\lambda + y \sin\lambda$$
$$d\eta = -(x \sin\lambda + y \cos\lambda) \sin\varphi$$

The rectangular coordinates X and Y are published by the International Latitude Service.